

A (reactive) lattice-gas approach to economic cycles

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Abstract

A microscopic approach to macroeconomic features is intended. A model for macroeconomic behavior under heterogeneous spatial economic conditions is reviewed. A birth-death lattice gas model taking into account the influence of an economic environment on the fitness and concentration evolution of economic entities is numerically and analytically examined. The reaction-diffusion model can be also mapped onto a high order logistic map. The role of the selection pressure along various dynamics with entity diffusion on a square symmetry lattice has been studied by Monte-Carlo simulation. The model leads to a sort of phase transition for the fitness gap as a function of the selection pressure and to cycles. The control parameter is a (scalar) "business plan". The business plan(s) allows for spin-offs or merging and enterprise survival evolution law(s), whence bifurcations, cycles and chaotic behavior.

Key words: econophysics, macroeconomy, evolution, external field, selection

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1 Introduction

Econophysics [1,2,3,4,5] has been centered mainly about developing continuous models, writing time dependent equations to describe fluxes, prices, assets, returns, risk, in order to obtain evolutions or stable states [6,7]. This leads to Langevin-like, and/or Fokker-Planck equations for price evolutions [8,9,10,11], even e.g. Black-Scholes equation for options [5,12] or simply partial distribution functions description [4,9]. This is line with Hamilton equation mechanistic approach which a classically trained physicist can accept as a basic approach! Quantum mechanistic ideas are sometimes coming in [12].

Another aspect has been to develop discrete, so called agent based, (microscopic) models¹, on lattices or not, on networks or not, in order to manipulate interactions and constraints in view of describing such characteristics as prices, returns, etc. and perform simulations. This is supposed to be interesting, according to economists, if the irrational or not behavior of agents is described as expected [14], even though physicists know that deterministic chaos exists. This "microscopic" approach is often coupled to data analysis, going from distribution functions to scaling exponents and universal laws for various features, including crashes [15].

In the MACRO economy, theoretical work on economic behaviors can also be thought of to relate to some "self-organized" features [15,16]. In the continuous time spirit, the variables of interest are sometimes GDP, immigration rate, wages, price, wealth, profit, taxes, capital growth,... The equations again are of the Langevin type and contain coupling parameters which are hardly measurable [17,18], and are mostly "ad hoc", even though based on so called "financial theory" expectations. One question exists whether for either rarely well quantified or abundantly quantified, in a statistical sense, many macroeconomic features can be described from a microscopic-like level, with understandable parameters [19]. This is in line with previous attempt to connect macroeconomy and econophysics [20].

From a macroeconomic behavior point of view, it is remarkable that there are spatio-temporal changing economic conditions, - and a variable space as well. E.g. after the Berlin wall opening, there was a sort of "physical volume, or available space" increase for economic entities, like for particles in a container. The gas particles are hereby called companies, but this is only a name for some economic variables. The company "degree of freedom" (particle efficiency f) can be coupled to an external field F . Beside the field, there is some selection pressure sel . Moreover as for a gas, economic entities are allowed to move. Also the f value of the new firms was considered to be obtained according to

¹ an exhaustive list of references should be too long; see for some interesting review Samanidou et al. [13]

various types of memories depending on the f of the company parents [21]. The algorithm of our toy-like model [21], a chemically reactive lattice gas, is recalled in Sect.2. It was aimed at investigating whether macroeconomic features, like so called economic cycles² [22,23] could be obtained or recovered from elementary rules with ("microscopic") interactions between entities characterized by some degree of freedom (their efficiency) coupled to a field which can be qualitatively considered an economic one. A so called business plan, based on a merging and spin-off creation alternative was considered for the enterprise concentration evolution. An *a priori law* is given for the efficiency evolution.

In section 3, we outline a few results, like the concentration and efficiency of entity evolutions, comparing some stochastic or not initiating conditions. From Monte-Carlo simulations it is observed that the model leads to a phase transition or self-organisation-like scenario as a function of the selection pressure. Some analytical results are also presented within a mean field approximation for the best adapted company under constant field conditions. A short conclusion is found in Sect. 4.

2 Model and algorithm sketch

Initially all firms are located at random positions on a lattice and receive a random f value. The algorithm, based on essentially three rules (R_i) is:

- (1) a firm (i) is picked;
- (2) (R_1) a survival probability

$$p_i = \exp(-sel|f_i - F|), \quad (1)$$

is checked against a uniformly distributed random number $r_i \in [0, 1]$.

If $r_i > p_i$ the firm is removed from the system;

- (3) (R_2) if the firm survived, then the firm is moved to a site of the nearest neighborhood, - if an empty place is found;
- (4) a random search is made for a partner in the nearest neighborhood of the new position. If found at the site j then (R_3)
- (5) either (with a probability b) the two firms merge, creating a new firm at the location of the first one, with a new fitness f_i while the second firm is eliminated;
- (6) or (with a probability $1 - b$) the two firms produce (a predetermined amount of) new (k) firms (spin-off's) with a (set of) fitness f_k in the Moore neighborhood (9 sites on a square symmetry lattice) of the first firm.

² <http://cepa.newschool.edu/het/schools/business.htm>

The three rules (R_i) of this model are the minimalistic set of rules that we can think of and yet yield a wealth of interesting evolutionary implications for the economy. Of course, there are many different variants at each stage. E.g. to pick up the firms, we have considered two cases: (i) either the pick is random (P_1), or (ii) the less adapted firm with respect to F is chosen (P_2).

Many variants can be imagined. One could allow for a predetermined or not number of searches for a move. The search about a free nearest neighbor site could be stochastic or deterministic. We only looked at a finite number of stochastic searches.

One could give or not new values of the fitness to the nearest neighbors. These values could be arbitrary, like in Bak-Sneppen original work [24], or might not be random. In our published work, the lattice was always with square symmetry, and the fitness evolution has always been driven by

$$f_i(t+1) = \frac{1}{2} [(f_i(t) + f_j(t)) + \text{sign}[0.5 - r]|f_i(t) - f_j(t)|], \quad (2)$$

where r is a random number in $[0,1]$.

The field, selection pressure, b can change in space and time; feedback or coupling between these parameters can be imposed; various boundary conditions can be imposed... N.B. The time is measured in Monte-Carlo steps (MCS). To complete a MCS one has to pick as many firms as there were at the beginning of that step.

3 A few results

3.1 Simulations

Results pertaining to two published cases [21,25] can be summarized; all pertain to an *a priori* $b=0.01$. Many facets can be revealed: the strength of the selection pressure is primordial, for reaching asymptotic values, but there are relatively well marked effects like

- (1) some equilibrium between births and deaths;
- (2) the concentration c_t can be maximal (P_1) or reach a finite value (P_2) (because in the latter, more Darwinian case, case the best adapted companies can never die);
- (3) the regions where field gradients exist are prone to instabilities at least [25] in case P_2 ;

- (4) the economic ("external") field implies stable or unstable density distributions (whence "cycles") ;
- (5) the diffusion process rule(s) are useful for invasion process, but are also relevant for replenishing abandoned regions (whence "cycles");
- (6) the average fitness is more or less slowly reached according to the selection pressure;
- (7) the "critical selection pressure" depends on the dynamics chosen
- (8) the business plan effect is very complex (see next section for analytical work about this).

3.2 Analytical approach

In fact, the possible events can be easily enumerated, since two kinds of birth-events, with respectively b and $1 - b$ probability, can be distinguished: (i) the number of f -states does not change: in this case the amount of companies existing on the net in the next MCS can be calculated by multiplying the existing number of companies in a given state by the appropriate survival probability; (ii) there is a change of the company state distribution. This is the case of company merging or spin-off creation. Considering all possible events, in a "mean field approximation", the evolution equation of the distribution function $N(t, f)$ can be written as the sum of two terms

$$N(t + 1, f) = H_1(c_t)p(f)N(t, f) + H_2(c_t)N(t, g(f)), \quad (3)$$

where the $H_1(c_t)$ and $H_2(c_t)$ polynomials (in c_t) containing coefficients depending on the various possible processes can be found elsewhere [26]. These polynomials mainly depend on the lattice symmetry and the number of spin-off's which are created. The function $g(f)$ describes the influence of the merging process on the distribution function $N(t, f)$. In the case of best adapted companies [26] ($f = F$), the evolution equation for $N(t, g(F))$ can be simplified and written as a logistic equation of high order. Notice that in this case the *sel* parameter is irrelevant.

In so doing we can focus on the b parameter effect. It can be shown that the system can (i) reach a one stable solution for $b > 0.45$; (ii) oscillate (Fig.1) with some characteristic time; (iii) display chaotic features for $b < 0.15$, including as usual "stability" windows. Especially interesting is the range $b \in [0.38; 0.45]$: damping properties are superposed to an oscillating behavior as seen from a study of a generalized Lyapunov exponent [26].

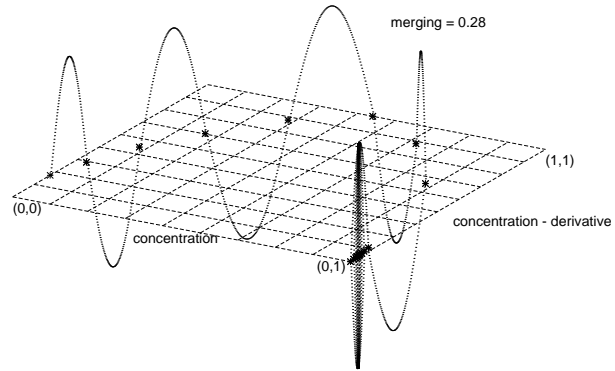
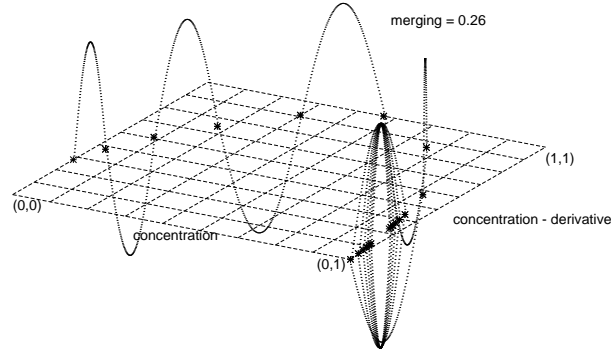
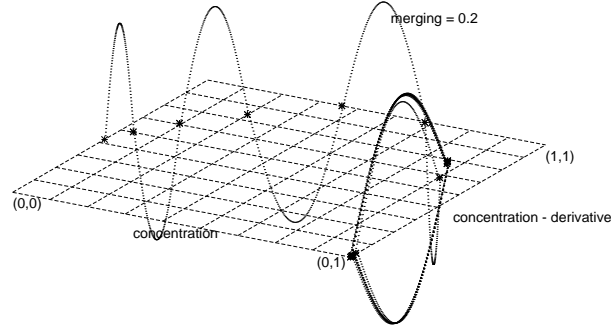


Fig. 1. Time evolution in parameter space (c_t, \dot{c}_t) of the $N(t, g(F))$ trajectory in the intermediary b regime characterized by cycles.

4 Conclusions

Changes in related *microeconomic* conditions may induce a change in the efficiency of a *macroeconomy*. For instance, a modification of traffic laws may distort the economic efficiency of a company. Likewise, social or cultural changes, perhaps driven by technological innovations, may induce an economic modification without any changes to the basic economic conditions themselves. The most important aspect of the above is to recognize that one does not need to stick to continuity evolution equations in order to describe such a macroeconomy evolution. We have thus presented a "death and birth reactive lattice gas process" along a microscopic physics like approach in order to describe a specific macroeconomy evolution. No need to say that the behavior, of a macroeconomy is of much greater complexity than as done here above. Fortunately many improvements are possible and needed.

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